

Reaction symmetry of irreversible reaction $nA + mB \rightarrow 0$

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We investigate the kinetics of irreversible reaction $nA + mB \rightarrow 0$. When nA particles and mB particles encounter on the same site, the reaction takes place. We classify the reaction into two classes, the symmetric and the asymmetric reactions. The symmetric reaction means both $nA + mB \rightarrow 0$ and $mB + nA \rightarrow 0$, while the asymmetric reaction means only $nA + mB \rightarrow 0$ for a given (n, m) pair. The kinetics of the reaction follows the fluctuation-dominated kinetics of $A + B \rightarrow 0$ for $N < N_c$, where N_c is the crossover $N (= n + m)$. For $N \geq N_c$, the kinetics follows a mean-field rate equation. For the asymmetric reaction, it was shown that N_c is 5. We numerically show that the reaction symmetry changes N_c in one dimension. We investigate the asymptotic scaling behaviors of density and various lengths characterizing the spatial organization of particles such as domain length and interparticle distance. Lengths exhibit much clear crossover to the mean-field region at the expected N_c of each reaction. From the scaling behavior of density and lengths, we show $N_c = 4$ for the symmetric reaction in one dimension. Therefore the symmetry in the reaction changes the upper critical dimension, which implies that the kinetics cannot be described by a single theory.

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I. INTRODUCTION

The irreversible two species reaction $A + B \rightarrow 0$ has been widely and intensively investigated among diffusion-limited reactions because of its rich kinetics and wide applications to various phenomena such as physics, chemistry, and biology [1–6]. The reaction instantaneously takes place when two particles of opposite species encounter on the same site. For homogeneous initial distributions with equal densities of A and B , $\rho_A(0) = \rho_B(0)$, the density $\rho(t)$ algebraically decays in time t . In dimensions higher than the upper critical dimensions ($d \geq d_c$), the kinetics follows a mean-field rate equation and ρ decays as $\rho \sim t^{-1}$ in time t . However for $d < d_c$, the random fluctuation of the initial number of two species increases as reaction proceeds, which leads to a segregation into A - and B -rich regions [4–6]. The fluctuation and the segregation develop in time. Therefore the reactions of two opposite species take place mainly at the boundaries of two adjacent segregated domains. As a result, the kinetics is fluctuation dominated for $d < d_c$ and cannot be described by any mean-field theory. For isotropic diffusions, ρ decays as $\rho \sim t^{-d/4}$ in $d < d_c = 4$ [4–9]. Subsequent studies showed that the kinetics depends on the motions, the mutual statistics of particles, and the interaction between opposite species [10–14].

For the higher-order reactions of two species, Cornell *et al.* [15] studied irreversible reaction $nA + mB \rightarrow 0$. When nA particles and mB particles encounter on the same site, the reaction takes place. For homogeneous initial conditions of densities $n\rho_A(0) = m\rho_B(0)$, ρ decays as $\rho \sim t^{-\alpha}$. It was conjectured from a microscopic argument that the kinetics depends on the number of particles engaged in the reaction, $N = n + m$. The kinetics is fluctuation dominated and the same type of the kinetics as that of $A + B \rightarrow 0$ for $N < N_c$. N_c is the crossover N over which the kinetics follows the mean-field rate equation,

$$d\rho_A(t)/dt = -k\rho_A^n\rho_B^m, \quad (1)$$

where k is the reaction rate. They found $N_c = 4$ in $d = 1$ and $N_c = 3$ in $d \geq 2$, respectively. Hence, the exponent α is given as $\alpha = d/4$ for $N < N_c$ and a mean-field value, $\alpha_{MF} = 1/(N - 1)$ for $N \geq N_c$. To systematically understand the fluctuation effect on the reaction $nA + mB \rightarrow 0$, Sasamoto *et al.* [16] applied the field-theoretic renormalization-group (RG) method to the reaction. It was shown that the kinetics depends on N rather than the each value of n and m . For instance, the kinetics for $N = 4$ is the same for $(n, m) = (2, 2)$ and $(1, 3)$ pairs. They found $d_c = 4/(N - 1)$ below which the kinetics belongs to the universality class of $A + B \rightarrow 0$, i.e., $\alpha = d/4$ in $d < d_c$.

However, intriguingly, the RG result for N_c contradicts with the prediction by Cornell [15] in low dimensions. In $d = 1$, the RG calculation predicts $N_c = 5$, whereas the argument of Cornell [15] gives $N_c = 4$. In $d \geq 2$, since the N_c of Cornell [15] cannot be obtained from the relation $d_c = 4/(N - 1)$, the N_c of the two studies happen to be the same as $N_c = 3$ in $d \geq 2$. As a result, Cornell [15] and Sasamoto [16] studied different reactions with different features. The discrepancy in $d = 1$ was discussed in the study of Sasamoto [16], but no clear explanations were proposed. In this paper we show that the disagreement comes from the symmetry of the reaction which gives nontrivial effects on the kinetics.

The previous studies [15, 16] concern only the reaction $nA + mB \rightarrow 0$ for a given (n, m) pair without the interchange of species in the reaction. For instance, for a given $(n, 1)$ pair, they allow only the reaction $nA + B \rightarrow 0$ and forbid the reaction $A + nB \rightarrow 0$. On the other hand, motivated by the argument of Cornell, we consider the reaction in which both $nA + mB \rightarrow 0$ and $mB + nA \rightarrow 0$ are allowed for a given (n, m) pair. We call the former reaction studied by Cornell [15] and Sasamoto [16] *asymmetric* reaction and the latter one *symmetric* reaction, respectively. In $d < d_c$, the segregation of particles leads to the difference of a reaction probability between the symmetric and asymmetric reactions in the do-

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mains of each species. For $nA+B\rightarrow 0$ reaction, the asymmetric reaction is suppressed in B -rich domains but enhanced in A -rich domains. However, the symmetric reaction occurs with an equal probability in the both species domains. As a result, the asymmetric reaction causes larger density fluctuations than those of the symmetric reaction. Therefore, it is expected that the kinetics depends on the reaction symmetry. The reaction symmetry has not been taken into account so far. Also in general, the symmetry is a very important feature in the study of critical phenomena [3]. Thus it is natural and important to investigate the role of the reaction symmetry to understand the kinetics of the reaction $nA+mB\rightarrow 0$.

In this paper, we study the kinetics of the symmetric and asymmetric reactions $nA+mB\rightarrow 0$ in one dimension. We show that the discrepancy between the previous studies [15,16] comes from the reaction symmetry rather than numerical errors. In the study of Cornell [15], their argument was based on the symmetric reaction, but they performed simulations for the asymmetric one only up to $N=4$ in $d=1$. Thus they cannot find out the contradiction between their predictions and numerical results. To see the difference between the two previous studies [15,16] and why the argument of Cornell holds only for the symmetric reaction and not for the asymmetric one, let us briefly review the argument of Cornell.

In the argument of Cornell [15], they consider the simplest reaction, $nA+B\rightarrow 0$ with $n>1$ and $N=n+1$. Since the segregation into A -rich and B -rich domains occurs as the reaction proceeds, they consider one B particle wandering in an A -rich domain of linear size ℓ in d dimensions. When the B particle visits a site occupied by more than nA particles, then the reaction takes place. To calculate N_c , they consider the probability $p_r(t)$ of the B particles having reacted with nA particles before time $t=\ell^2$. One can write $p_r(t)$ as $p_r=p_r^{MF}+\Delta p_r$, where p_r^{MF} is the mean-field p_r and Δp_r is the fluctuation of p_r due to the density fluctuations. $p_r(t)$ is given as $p_r(t)=N_v(t)p_A^n(t)$. $N_v(t)$ is the number of distinct sites visited by B particle during time t and $p_A(t)$ is the probability of finding an A particle at time t in the domain. N_v scales as $t^{1/2}$ in $d=1$, $t/\ln t$ in $d=2$, and t in $d=3$ [15]. Since p_A is the sum of the mean-field part $p_A^{MF}=\rho_A$ and the fluctuation part $\Delta p_A\sim\ell^{-d/2}$, p_r^{MF} and Δp_r are given as $p_r^{MF}=N_v\rho_A^n$ and $\Delta p_r\sim N_v\ell^{-dn/2}$, respectively. Therefore, if Δp_r vanishes for $t\rightarrow\infty$, the kinetics follows the mean-field rate [Eq. (1)]. With the scaling of N_v , Δp_r scales as $t^{(2-n)/4}$ in $d=1$, $t^{(2-n)/2}/\ln t$ in $d=2$, and $t^{(4-dn)/4}$ in $d=3$, respectively. Since Δp_r vanishes as $t\rightarrow\infty$ for $n>2$ in $d=1$ and $n\geq 2$ in $d\geq 2$, one finds $N_c=4$ in $d=1$ and $N_c=3$ in $d\geq 2$, respectively.

However, the different results should be obtained if the argument of Cornell was applied to the case in which nA -particles react with a B particle in a B -rich domain with the B -particle density ρ_B . Since the number of B -particles $N_B(\ell)$ in a B -rich domain of size ℓ is $N_B(\ell)=\ell^d\rho_B$, the p_r in a B -rich domain should be $\ell^d\rho_B$ times larger than p_r in a A -rich domain. Hence one finds $p_r=\ell^d\rho_B N_v\rho_A^n$ in a B -rich domain. Since the fluctuation Δp_B is also given as $\Delta p_B\sim\ell^{-d/2}$, Δp_r in B -rich domains scales as $\Delta p_r\sim N_v\ell^{-d(n-1)/2}$. N_c in B -rich domains should thus be larger than N_c in A -rich domain by one. In $d=1$, N_c is 5 which contradicts with $N_c=4$ in A -rich domains. Therefore, the argument of Cornell

does not correctly describe the kinetics of the asymmetric reaction $nA+B\rightarrow 0$, which explains why the predictions of Cornell are different from those of the RG calculation [16].

Instead, if one consider the symmetric reaction which allows both $nA+B\rightarrow 0$ and $A+nB\rightarrow 0$ reaction for a given n , Δp_r is given as $N_v\rho_A^n$ in A -rich domains and $N_v\rho_B^n$ in B -rich domains, respectively. Since p_A^n and p_B^n are symmetric, the argument of Cornell consistently describes the kinetics in both species domains. Therefore, the argument of Cornell [15] seems to be valid for the symmetric reaction rather than the asymmetric one.

In this paper, we investigate the effect of the reaction symmetry on the kinetics and numerically confirm that N_c varies according to the reaction symmetry in one dimension. Using three different models, we show that N_c is 4 for the symmetric reaction and 5 for the asymmetric one in $d=1$. $N_c=4$ for the symmetric reaction can be explained by the argument of Cornell [15]. We also show that the scaling behaviors of the various lengths characterizing the spatial organization of particles are also good measures to decide N_c in addition to the particle density.

In Sec. II, we introduce three different models, an interacting bosonic model, and two models with hard-core (HC) interaction between particles. We present Monte Carlo simulation results in Sec. III and finally conclude with summary in Sec. IV.

II. MODELS

We introduce three different models in one dimension; a bosonic model with on-site attractive interaction, HC model, and HC model with position interchange between opposite species (HC-interchange model). Since HC interaction is irrelevant to the kinetics for isotropic diffusions [10,11], we expect that the three models exhibit the same kinetics. The aim of studying three different models is the crossover check of asymptotic behavior.

In the bosonic model with on-site interaction, one randomly select a site i . If the selected site is occupied by n_i particles of the same species, n_i^λ particles simultaneously hop to the randomly selected nearest neighboring site. Since the number of hopping particles is suppressed by the exponent $\lambda(<1)$, there is the attractive interaction between particles occupying the same site. The case of $\lambda=1$ corresponds to noninteracting boson model. If the site i is occupied by n_A A particles and n_B B particles with $n_i=n_A+n_B$, n_A^λ/n_i A particles and n_B^λ/n_i B particles hop to the same target site. If the target site is occupied by both species after the hopping, the reaction takes place as follows. For the symmetric reaction, if the number of A particles (n_A) and B particles (n_B) satisfy either $n_A\geq n$ and $n_B\geq m$ or $n_A\geq m$ and $n_B\geq n$, then the reaction takes places until the condition is not satisfied. For the asymmetric reaction, the reaction occurs only if $n_A\geq n$ and $n_B\geq m$. This model is the generalized bosonic model of $N=2$ case [14].

In HC model, the number of particles at a site is at most one due to HC interaction between all particles. If a randomly selected site is occupied by a particle, then this particle attempts to hop to the randomly selected nearest neigh-

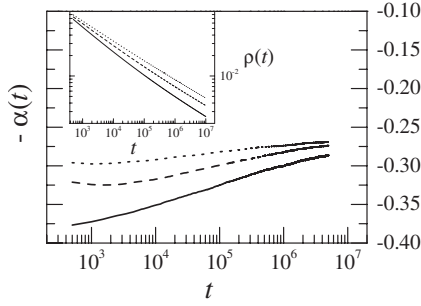


FIG. 1. Symmetric $2A+B\rightarrow 0$: plot of the effective exponent $\alpha(t)$. From top to bottom, the dotted, dashed, and solid lines correspond to HC model, HC-interchange model, and bosonic model with $\lambda=0$, respectively. Inset shows $\rho(t)$.

boring site. If the target site is empty, the hopping is accepted. Otherwise, the attempt is rejected. After the hopping attempt regardless of its result, the reaction takes place in the following cases. One considers $N-1$ nearest neighboring sites from the position of the selected particle in the direction of the target site. If all N sites including the selected site are occupied, then the reaction occurs according to the symmetry. In the symmetric reaction, if the number of A particles (n_A) and B particles (n_B) among N particles satisfy either $n_A=n$ and $n_B=m$ or $n_A=m$ and $n_B=n$, then the reaction takes places. In the asymmetric reaction, the reaction occurs only if $n_A=n$ and $n_B=m$.

The dynamics of HC-interchange model is the same as those of HC model except following cases. If the selected particle attempts to hop to the target site occupied by a particle of opposite species, then the two particles interchange their positions such as $AB\rightarrow BA$.

III. MONTE CARLO SIMULATIONS

A. Symmetric reaction

We simulate the three models on one-dimensional lattice with the periodic condition. With homogeneous initial conditions of equal densities of A and B , $\rho_A(0)=\rho_B(0)$, we perform simulations on a ring of size L for various N up to 6. For the bosonic model, it was shown that the kinetic is not changed by $\lambda(<1)$ [14], and we set $\lambda=0$ in simulations. Since we numerically confirm the same results for various (n,m) pairs of a given N , we present the results of $N=n+1$ case for simplicity, i.e., $nA+B\rightarrow 0$ and $A+nB\rightarrow 0$ reaction. For the symmetric reaction, the kinetics is fluctuation dominated for $N<4$ and follows the mean-field rate [Eq. (1)] for $N\geq 4$ according to the argument of Cornell [15]. Hence, the exponent α is $\alpha=1/4$ for $N<4$ and $\alpha_{MF}=1/(N-1)$ for $N\geq 4$. Since the $N=2$ case has been studied extensively [6,8,9], we consider the cases of $N>2$.

We set $\rho(0)=1.0$ for HC models and $\rho(0)=0.5$ for the bosonic model for all N values. For $N=3$, we perform Monte Carlo simulations with L up to 5×10^6 . We run simulations up to 10^7 Monte Carlo time steps and average $\rho(t)$ over from 100 to 400 independent runs. Figure 1 shows $\rho(t)$ of $N=3$ and the effective exponent $\alpha(t)$ defined as $-\alpha(t)=\ln[\rho(bt)/\rho(t)]/\ln b$, with $b=2$. As shown, $\alpha(t)$'s of three

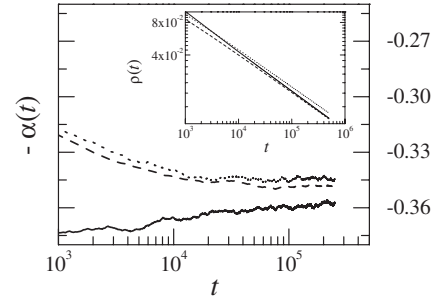


FIG. 2. Symmetric $3A+B\rightarrow 0$: plot of the effective exponent $\alpha(t)$. From top to bottom, the dotted, dashed, and solid lines correspond to HC model, HC-interchange model, and bosonic model with $\lambda=0$, respectively. Inset shows $\rho(t)$.

models simultaneously approach to the expected value $1/4$ very slowly. We estimate α from the scaling plot of ρt^α against t (not shown). We obtain $\alpha=0.270(5)$ for HC model, $0.275(5)$ for HC-interchange model and $0.285(5)$ for the boson model with $\lambda=0$, respectively. Hence three models exhibit the same kinetics as that of $A+B\rightarrow 0$ reaction as expected.

On the other hand, from $N=4$, the kinetics is expected to be described by the mean-field rate [Eq. (1)]. With the system size $L=10^5$, we measure $\rho(t)$ up to $t=5\times 10^5$ for $N=4$. Figure 2 shows ρ and $\alpha(t)$ of $N=4$. The HC models approach to the mean-field scaling region faster than the bosonic model does. From the scaling plots, we estimate $\alpha=0.345(5)$ for HC model, $0.348(5)$ for HC-interchange model and $0.359(5)$ for the boson model with $\lambda=0$, respectively. Although $\alpha(t)$ of all models are still larger than the expected mean-field value $1/3$ of $N=4$, we are convinced that the kinetic follows the mean-field rate [Eq. (1)].

For more concrete evidence that the kinetics of $N=4$ follows the mean-field rate [Eq. (1)], we investigate the spatial organization of particles. For the fluctuation-dominated kinetics, the distance between adjacent segregated domains and the distance between adjacent particles in a single domain exhibit different scaling behaviors [8]. However, for the mean-field kinetics, particles are uniformly distributed without the segregation. Hence, the spatial organization of particles is determined by a single length which scales as $1/\rho$. Therefore, one can determine the type of the kinetics by investigating the spatial organization of particles which is characterized by various lengths.

We define various lengths as follows. The average length of a single species domains (ℓ) is defined as the distance between the first particles of two adjacent opposite species domains. The length ℓ_{AA} (ℓ_{BB}) and ℓ_{AB} are defined as the interparticle distance between two adjacent particles of the same species and of opposite species, respectively [8]. The lengths algebraically increase in time t as

$$\ell_{AA} \sim t^{1/Z_{AA}}, \quad \ell_{AB} \sim t^{1/Z_{AB}}, \quad \ell \sim t^{1/Z}. \quad (2)$$

For $A+B\rightarrow 0$ reaction, the lengths scale in time as $\ell \sim t^{1/2}$, $\ell_{AA} \sim t^{1/4}$, and $\ell_{AB} \sim t^{3/8}$. However, since particles are uniformly distributed in mean-field region, all lengths should

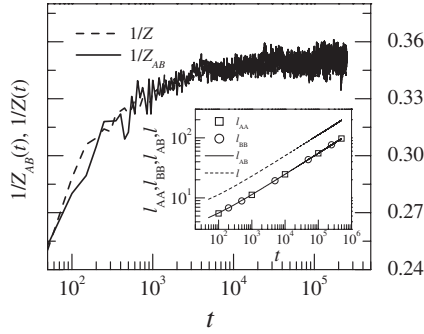


FIG. 3. Symmetric $3A+B \rightarrow 0$: plot of the effective exponent $1/Z_{AB}$ and $1/Z(t)$ of the bosonic model with $\lambda=0$. Inset shows various lengths.

scale as $1/\rho(t)$. Hence, we expect that all lengths scales as $t^{1/3}$ for $N=4$.

For the measurement of various lengths, it is not clear to define the domain length ℓ in HC models because particles of the one species remain inside a domain of the other species for $N>2$. Instead, we use the bosonic model with $\lambda=0$ rather than HC models. In the measurement, we measure the distances between the sites occupied by particles of the same species and neglect the sites occupied by both species.

Figure 3 shows the lengths and the effective exponent defined as $1/Z(t) = \ln[\ell(bt)/\ell(t)]/\ln b$, with $b=2$, and similarly for the others. As shown the inset of Fig. 3, three lengths, ℓ_{AA} , ℓ_{BB} , and ℓ_{AB} overlap. The domain length ℓ is larger than the interparticle distance by a constant. As shown in the main plot of Fig. 3, two effective exponents, $1/Z_{AB}$ and $1/Z$, overlap one another so that all lengths exhibit the same scaling behavior. We estimate $1/Z$ from the scaling plot of $\ell/t^{1/Z}$ and $1/Z_{AB}$ similarly (not shown). We estimate $1/Z=0.348(5)$ and $1/Z_{AB}=0.350(5)$ which are nearly the same as the α of the bosonic model (see Fig. 2). Hence, all lengths scales as $t^{1/3} \sim 1/\rho$ for $N=4$. As a result, we conclude that the kinetics of $N=4$ follows the mean-field rate [Eq. (1)].

For $N>4$, we perform simulation up to $t=5 \times 10^5$ with $L=10^5$ and confirm that the kinetics follows the mean-field rate [Eq. (1)]. In Fig. 4, we present α of the three models and $1/Z$'s of the boson model with $\lambda=0$ for various N . We estimate α and $1/Z$ using the scaling plot of ρt^α and $\ell t^{-1/Z}$ against t (not shown). Similarly, we estimate $1/Z_{AA}$, $1/Z_{BB}$, and $1/Z_{AB}$. As shown, ρ and all lengths exhibit the same scaling behaviors as those of $A+B \rightarrow 0$ for $N<4$. Hence, the kinetics is fluctuation dominated for $N<4$. On the other hand, for $N \geq 4$, the exponents of the density and all lengths collapse on the mean-field line $\alpha_{MF}=1/(N-1)$. As a result, the kinetics follows the mean-field rate [Eq. (1)] for $N \geq 4$. Our numerical results agree well with the predictions of Cornell [15] for the symmetric reaction.

B. Asymmetric reaction

Sasamoto *et al.* [16] studied $nA+mB \rightarrow 0$ reaction using field-theoretic RG analysis and numerically confirmed their predictions. In the RG analysis, they studied the master equation $\partial|P(t)\rangle/\partial t = -\hat{H}|P(t)\rangle$ with the evolution operator \hat{H} defined as

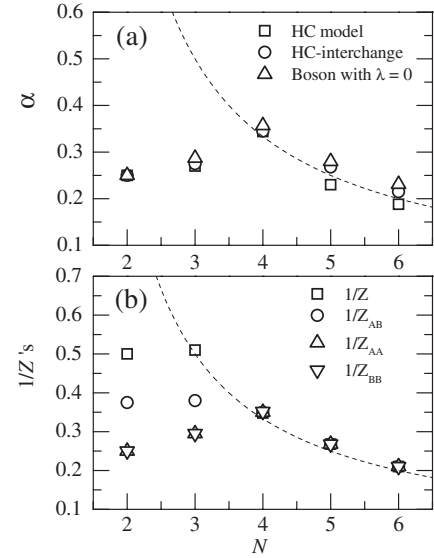


FIG. 4. Symmetric $nA+B \rightarrow 0$: plot of (a) α and (b) $1/Z$'s of the boson model with $\lambda=0$ against N . The dashed line corresponds to $\alpha_{MF}=1/(N-1)$.

$$\hat{H} = D \sum_{\langle ij \rangle} \{ (a_j^\dagger - a_i^\dagger)(a_j - a_i) + (b_j^\dagger - b_i^\dagger)(b_j - b_i) \} + k \sum_i (a_i^\dagger)^m (b_i^\dagger)^n a_i^m b_i^n, \quad (3)$$

where a_i^\dagger , a_i and b_i^\dagger , b_i are the bosonic operators for each species and satisfy a commutation relation, $[a_i, a_i^\dagger] = [b_i, b_i^\dagger] = \delta_{ij}$. D and k are the diffusion and reaction rates, respectively. Since \hat{H} is asymmetric for the interchange of species, the reaction $mA+nB \rightarrow 0$ is not allowed for a given (n, m) pair. Hence, the reaction is asymmetric for $n \neq m$. Intriguingly, for $n=m$, \hat{H} is symmetric for the interchange of species, and thus the reaction is symmetric. From the RG analysis, they found $d_c=4/(N-1)$, and the kinetics follows the mean-field rate [Eq. (1)] for $d \geq d_c$. For $d < d_c$, the kinetics belongs to the class of $A+B \rightarrow 0$ reaction.

In the previous study [16], they numerically confirmed the RG predictions using a HC model with N up to 6 in one dimension. For the completeness of the present paper, we present our numerical results for the asymmetric reaction $nA+B \rightarrow 0$ with N up to 6. In addition, as supplementary evidences, we also present the scaling behaviors of various lengths according to N . With homogeneous initial conditions of densities, $n\rho_A(0)=\rho_B(0)$, we perform simulations up to $t=5 \times 10^6$ with L up to 3×10^6 . We estimate α and $1/Z$'s from the same scaling plots as those of the symmetric cases.

In Fig. 5, we plot α of the three models and $1/Z$'s of various lengths of the bosonic model with $\lambda=0$. As shown, the $1/Z$'s clearly collapse on the mean-field line $\alpha_{MF}=1/(N-1)$ for $N \geq 5$. Hence, the kinetic follows the mean-field rate [Eq. (1)] for $N \geq 5$ as expected. On the other hand, for $N<5$, $1/Z$'s are different and α deviate from the mean-field line. The values of all exponents are consistent with those of $A+B \rightarrow 0$ reaction. Hence, the kinetics is fluctuation dominated and belongs to the class of $A+B \rightarrow 0$ reaction for

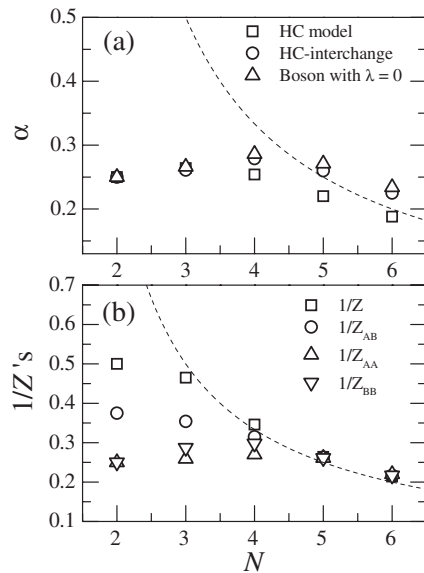


FIG. 5. Asymmetric $nA+B \rightarrow 0$: plot of (a) α and (b) $1/Z$'s of the boson model with $\lambda=0$ against N . The dashed line corresponds to $\alpha_{MF}=1/(N-1)$.

$N < 5$. However, as N increases to $N_c=5$, all exponents become to deviate from the expected values. It means that systems suffer from very strong corrections to the scaling as N increases to $N_c=5$. All our numerical results also agree well with the RG predictions.

IV. SUMMARY

In summary, we investigate the symmetry of irreversible reaction $nA+mB \rightarrow 0$. We classify the reaction into two classes, symmetric and asymmetric reaction according to the symmetry of the interchange of species in the reaction. While the symmetric reaction includes both $nA+mB \rightarrow 0$ and $mA+nB \rightarrow 0$, the asymmetric reaction considers only $nA+mB \rightarrow 0$ for a given (n, m) pair.

In one dimension, the kinetics crosses over from the fluctuation-dominated kinetics of $A+B \rightarrow 0$ reaction to the mean-field one at different N_c according to the symmetry. The number N is the number of particles engaged in the reaction, i.e., $N=n+m$, and N_c is the crossover N over which the kinetics follows the mean-field rate [Eq. (1)]. For the asymmetric reaction, the kinetics is described by the RG analysis [16] which shows $N_c=5$ in one dimension. On the other hand, the kinetics of the symmetric reaction can be

described by the argument of Cornell [15] which shows $N_c=4$ in one dimension. In $d \geq 2$, N_c is 3 regardless of the symmetry. However, since the N_c of Cornell cannot be obtained from the relation $d_c=4/(N-1)$, the N_c of the two studies happen to be the same as $N_c=3$ in $d \geq 2$. Therefore the symmetry in the reaction changes the upper critical dimension, which implies that the kinetics cannot be described by a single theory.

We numerically confirm $N_c=5$ for the symmetric reaction and $N_c=4$ for the asymmetric reaction, respectively. In addition to the scaling behavior of density, we investigate the scaling behaviors of various lengths characterizing the spatial organization of particles. We numerically confirm that the scaling behaviors of the lengths are the same for $N \geq N_c$ and different for $N < N_c$. The scaling behaviors of the lengths allow one to estimate N_c clearly.

For the symmetric reaction, we mainly understand the kinetics via simulations and heuristic arguments. Hence, to understand the symmetric reaction systematically, systematic theoretical analyses are needed such as the field-theoretic RG analysis or an analysis of the phase portrait [17]. The analysis of the phase portrait was recently introduced by Elgart and Kamenev [17] as a classification scheme of phase transitions in reaction-diffusion models. In the analysis, one can extract the information about the universality class of a given reaction-diffusion system by analyzing the topology of the phase portrait which consists of special trajectories satisfying a certain condition for the Hamiltonian H of a given reaction such as Eq. (3). It was shown that the systems belonging to the same universality class share the same topology of the phase portrait [17]. Hence, for the reaction $nA+mB \rightarrow 0$, it is expected that the phase portrait exhibits a distinct topology according to the reaction symmetry. Since the analysis of the phase portrait is much simpler way to understand the critical behavior of reaction-diffusion systems, it is needed to analyze the phase portrait of the reaction $nA+mB \rightarrow 0$ as the first step toward the systematic theoretical understanding of the role of the reaction symmetry before going to the full RG analysis.

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